विचार संगम

भारतीय दर्शन, विज्ञान, साहित्य-कला और तकनीकी के विशेष संदर्भ में

सम्पादक
डॉ. दिनेश चन्द्र शर्मा

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FUNCTIONS

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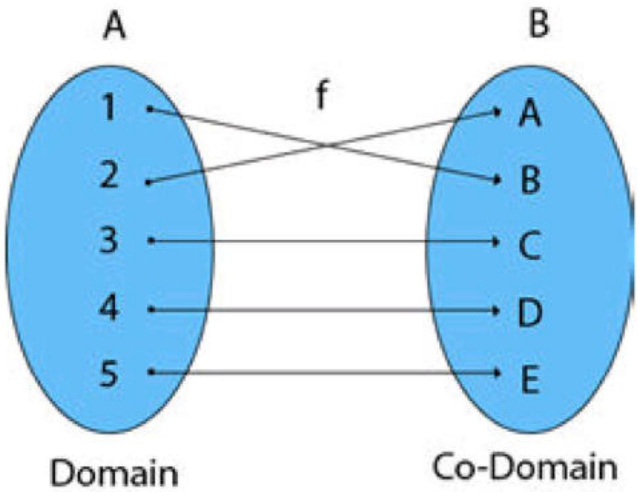
It is a mapping in which every element of set A is uniquely associated at the element with set B. The set of A is called Domain of a function and set of B is called Co domain.



Domain, Co-Domain, and Range of a Function

Domain of a Function :- Let f be a function from P to Q. The set P is called the domain of the function f.

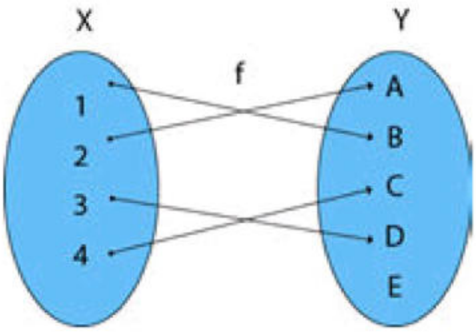
Co-Domain of a Function :- Let f be a function from P to Q. The set Q is called Co-domain of the function f.

Range of a Function :- The range of a function is the set of picture of its domain. In other words, we can say it is a subset of its co-domain. It is denoted as f (domain).

1. If $f : P \rightarrow Q$, then $f(P) = \{f(x): x \in P\} = \{y: y \in Q \mid \exists\ x \in P$, such that $f(x) = y\}$.

Example: Find the Domain, Co-Domain, and Range of function.

1. Let x = {1, 2, 3, 4}
2. y = {a, b, c, d, e}
3. f = {(1, b), (2, a), (3, d), (4, c)



Solution:

Domain of function: {1, 2, 3, 4}

Range of function: {a, b, c, d}

Co-Domain of function: {a, b, c, d, e}

Functions as a Set :-

If P and Q are two non-empty sets, then a function f from P to Q is a subset of P x Q, with two important restrictions

1. $\forall$ a $\in$ P, (a, b) $\in$ f for some b $\in$ Q
2. If (a, b) $\in$ f and (a, c) $\in$ f then b = c.

Note1 :- There may be some elements of the Q which are not related to any element of set P.

2. Every element of P must be related with at least one element of Q.

Example1 :- If a set A has n elements, how many functions are there from A to A?

Solution :- If a set A has n elements, then there are n^n functions from A to A.

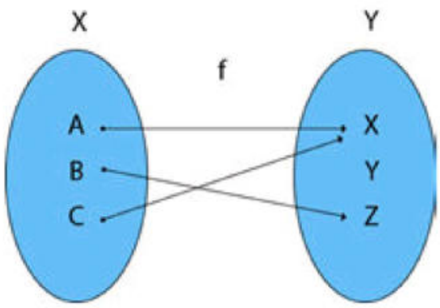
Representation of a Function :-

The two sets P and Q are represented by two circles. The function f: P ? Q is represented by a collection of arrows joining the points which represent the elements of P and corresponds elements of Q

Example1 :-

1. Let X = {a, b, c} and Y = {x, y, z} and f: X $\rightarrow$ Y such that
2. f = {(a, x), (b, z), (c, x)}

Then f can be represented diagrammatically as follows



Example2 : Let X = {x, y, z, k} and Y = {1, 2, 3, 4}. Determine which of the following functions. Give reasons if it is not. Find range if it is a function.

a. f = {(x, 1), (y, 2), (z, 3), (k, 4)

b. g = {(x, 1), (y, 1), (k, 4)

c. h = {(x, 1), (x, 2), (x, 3), (x, 4)

d. l = {(x, 1), (y, 1), (z, 1), (k, 1)}

e. d = {(x, 1), (y, 2), (y, 3), (z, 4), (z, 4)}.

Solution:

1. It is a function. Range (f) = {1, 2, 3, 4}

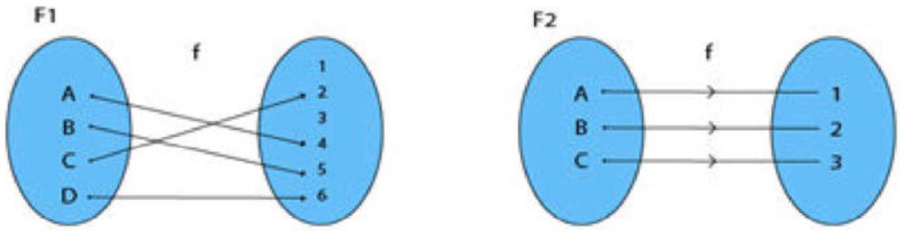
2. It is not a function because every element of X does not relate with some element of Y i.e., Z is not related with any element of Y.

3. h is not a function because h (x) = {1, 2, 3, 4} i.e., element x has more than one image in set Y.

4. d is not a function because d (y) = {2, 3} i.e., element y has more than image in set Y.

Types of Functions

1. Injective (One-to-One) Functions: A function in which one element of Domain Set is connected to one element of Co-Domain Set.

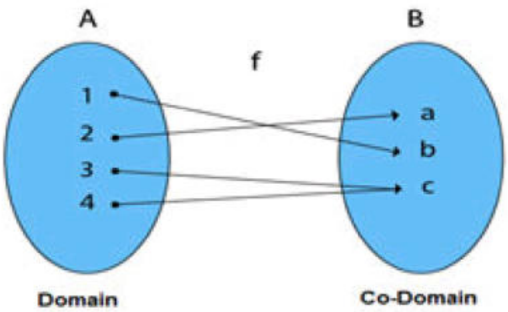


F1 and F2 show one to one Function

2. Surjective (Onto) Functions :- A function in which every element of Co-Domain Set has one pre-image.

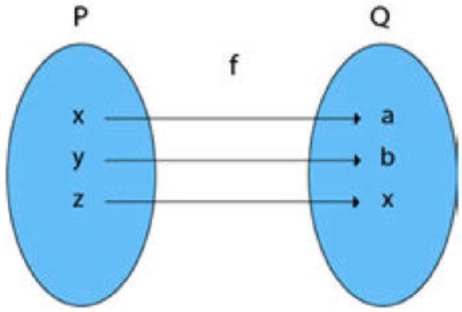
Example :- Consider, A = {1, 2, 3, 4}, B = {a, b, c} and f = {(1, b), (2, a), (3, c), (4, c)}.

It is a Surjective Function, as every element of B is the image of some A



Note: In an Onto Function, Range is equal to Co-Domain.

3. Bijective (One-to-One Onto) Functions :- A function which is both injective (one to - one) and surjective (onto) is called bijective (One-to-One Onto) Function.



Example :-

1. Consider P = {x, y, z}
2. Q = {a, b, c}
3. and f: P ? Q such that
4. f = {(x, a), (y, b), (z, c)}

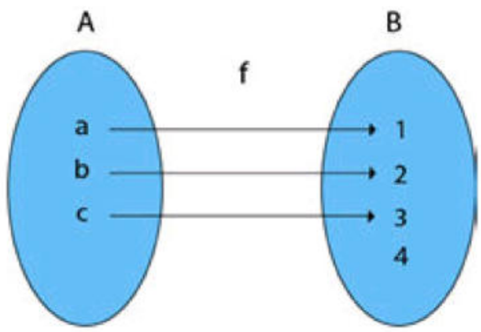
The f is a one-to-one function and also it is onto. So it is a bijective function.

4. Into Functions :- A function in which there must be an element of co-domain Y does not have a pre-image in domain X.

Example :-

1. Consider, A = {a, b, c}
2. B = {1, 2, 3, 4}   and f: A ? B such that
3. f = {(a, 1), (b, 2), (c, 3)}
4. In the function f, the range i.e., {1, 2, 3} ? co-domain of Y i.e., {1, 2, 3, 4}

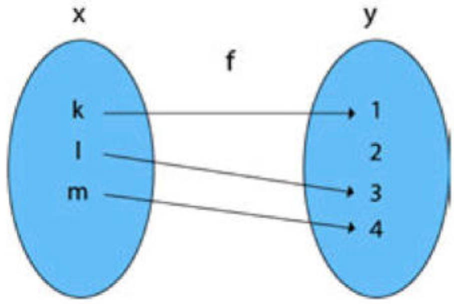
Therefore, it is an into function



5. One-One Into Functions :- Let f: X → Y. The function f is called one-one into function if different elements of X have different unique images of Y.

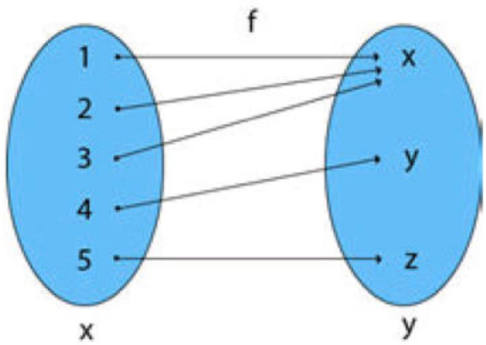
Example :-

1. Consider, X = {k, l, m}
2. Y = {1, 2, 3, 4} and f: X ? Y such that
3. f = {(k, 1), (l, 3), (m, 4)}
The function f is a one-one into function



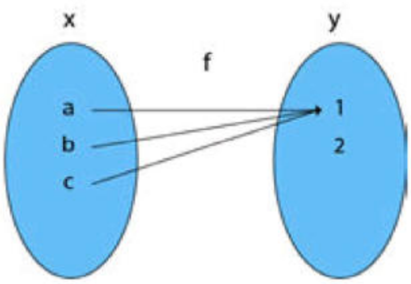
6. Many-One Functions :- Let f: X → Y. The function f is said to be many-one functions if there exist two or more than two different elements in X having the same image in Y.

Example :-
1. Consider X = {1, 2, 3, 4, 5}
2. Y = {x, y, z} and f: X ? Y such that
3. f = {(1, x), (2, x), (3, x), (4, y), (5, z)}
The function f is a many-one function



7. Many-One Into Functions :- Let f: X → Y. The function f is called the many-one function if and only if is both many one and into function.

Example :-
1. Consider X = {a, b, c}
2. Y = {1, 2} and f: X → Y such that
3. f = {(a, 1), (b, 1), (c, 1)}
As the function f is a many-one and into, so it is a many-one into function.

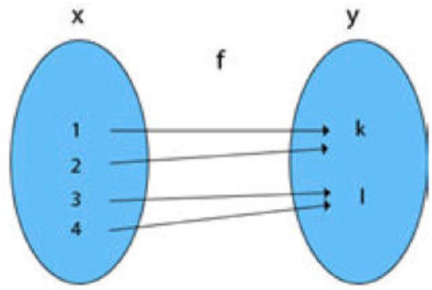


8. Many-One Onto Functions :- Let f: X → Y. The function f is called many-one onto function if and only if is both many one and onto.

Example:-

1. Consider X = {1, 2, 3, 4}
2. Y = {k, l} and f: X → Y such that
3. f = {(1, k), (2, k), (3, l), (4, l)}

The function f is a many-one (as the two elements have the same image in Y) and it is onto (as every element of Y is the image of some element X). So, it is many-one onto function



Compositions of Functions

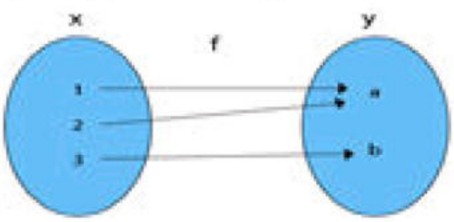
Consider functions, f: A → B and g: B → C. The composition of f with g is a function from A into C defined by (gof) (x) = g [f(x)] and is defined by gof.

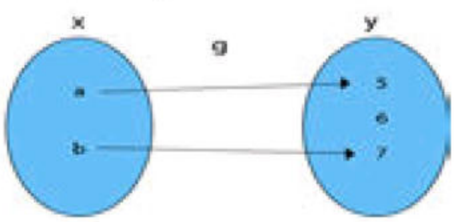
To find the composition of f and g, first find the image of x under f and then find the image of f (x) under g.

Example1 :-

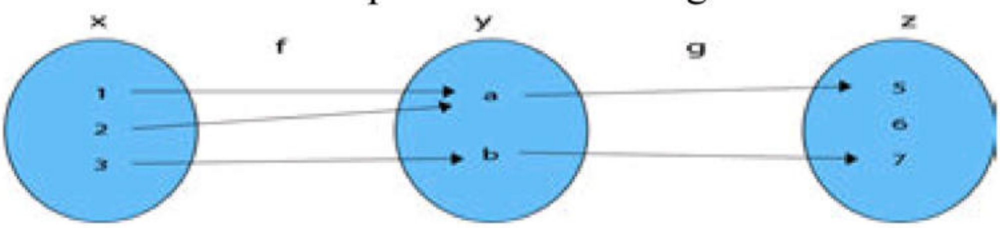
1. Let X = {1, 2, 3}
2. Y = {a, b}
3. Z = {5, 6, 7}.

Consider the function f = {(1, a), (2, a), (3, b)} and g = {(a, 5), (b, 7)} as in figure. Find the composition of gof.





Solution :- The composition function gof is shown in fig:



(gof) (1)=g[f (1)] = g (a) = 5, (gof) (2) = g [f (2)]=g(a) = 5

(gof) (3) = g [f (3)] = g (b) = 7.

Example2 :- Consider f, g and h, all functions on the integers, by f (n) =n2, g (n) = n + 1 and h (n) = n - 1.

Determine (i) hofog (ii) gofoh (iii) fogoh.

Solution :-

(i) hofog (n) = n + 1,

hofog (n + 1) = $(n+1)^2$

h $[(n+1)^2]$ = $(n+1)^2 - 1 = n^2 + 1 + 2n - 1 = n^2 + 2n$.

(ii) gofoh (n) = n - 1, gof (n - 1) = $(n-1)^2$

g $[(n-1)^2]$ = $(n-1)^2 + 1 = n^2 + 1 - 2n + 1 = n^2 - 2n + 2$.

(iii) fogoh (n) = n - 1

fog (n - 1) = (n - 1) + 1

f (n) = n^2.

Note :-

* If f and g are one-to-one, then the function (gof) (gof) is also one-to-one.

* If f and g are onto then the function (gof) (gof) is also onto.

* Composition consistently holds associative property but does not hold commutative property.

Mathematical Functions

The following are the functions which are widely used in computer science.

1. Floor Functions :- The floor function for any real number x is defined as f (x) is the greatest integer 1 less than or equal to x. It is denoted by [x].

Example :- Determine the value of

(i) [3. 5] (ii) [-2.4] (iii) [3. 143].

Solution :-

(i) [3 . 5] = 3

(ii) [-2 .4] = -3

(iii) [3. 143] = 3

2. Ceiling Functions :- The ceiling function for any real number x is defined as h (x) is the smallest integer greater than or equal to x. It is denoted by [x].

Example :- Determine the value of

(i)[3. 5] (ii) [-2.4] (iii) [3. 143].

Solution :-

(i)[3. 5] = 4

(ii) [-2 .4] = -2

(iii) [3. 143] = 4.

3. Remainder Functions :- The integer remainder is obtained when some a is divided by m. It is denoted by a (MOD m). We can also define it as, a (MOD m) is the unique integer t such that a = Mq + t. Here q is quotient $0 \leq r < M$.

Example :- Determine the value of the following:

(i) 35 (MOD 7) (ii) 20 (MOD 3) (iii) 4 (MOD 9)

Solution :-

(i) 35 (MOD 7) = 0

(ii) 20 (MOD 3) = 2

(iii) 4 (MOD 9) = 4

4. Exponential Functions :- Consider two sets A and B. Let $A = B = I_+$ and also let $f: A \rightarrow B$ be defined by $f(n) = k^n$. Here n is a +ve integer. The function f is called the base k exponential function.

Note1:- kt= k. k. k.......k (t times).

2: $k^0=1, k^{-M} = \dfrac{1}{k^M}$

3. For rational number, a/b, the exponential function is

$$k^{a/b} = \sqrt[b]{k^a} = \left(\sqrt[b]{k}\right)^M$$

Example :- Determine the value of the following:

(i) 10^3 (ii) $5^{1/2}$ (iii) 3^{-5}

Solution :-

$10^3 = 10.\ 10.\ 10 = 1000$

$5^{1/2} = 2.23607$

$$3^{-5} = \frac{1}{3^5} = \frac{1}{3.3.3.3.3} = \frac{1}{243}$$

5. Logarithmic Functions :- Consider two sets A and B. Let A = B = R (the set of real numbers and also let $f\_n: A \rightarrow B$ be defined for each positive integer n > 1 as $f_n(x) = \log_n(x)$ the base n of x.

Note1:- $k = \log_n x$ and n^k are equivalent.

2. For any base n, $\log_n 1=0$ as $n^0=1$.

3. For any base n, $\log_n n=1$ as $n^1=n$.

Example :- Determine the value of the following:

(i) $\log_2?16$ (ii) $\log_2 100$ (iii) log_2 0.001.

Solution :-

(i) $\log_2 16 = 4$ as $2^4=16$.

(ii) $\log_2 100 = 6$ as $2^6= 64$ but $2^7=128$ which is greater

(iii)$\log_2 0.001 = -9$ as $2^{-9} = \frac{1}{512}$ but $2^{-10} = \frac{1}{1024}$ which is greater.

Algorithms and Functions

Algorithm :- An algorithm is a step-by-step method for solving some problem.

Algorithms generally have the following characteristics :-

1. Input :- The algorithm receives input. Zero or more quantities are externally supplied.

2. Output :- The algorithm produces output. At least one quantity is produced.

3. Precision :- The steps are precisely stated. Each instruction is clear and unambiguous.

4. Feasibility :- It must be feasible to execute each instruction.

5. Flexibility :- It should also be possible to make changes in the algorithm without putting so much effort on it.

6. Generality :- The algorithm applies to a set of inputs.

7. Finiteness :- Algorithm must complete after a finite number of instruction have been executed.